

ON THE SUCCESS OF NETWORK INFERENCE USING A MARKOV ROUTING MODEL

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ABSTRACT

In this paper we discuss why a simple network topology inference algorithm based on network co-occurrence measurements and a Markov random walk model for routing enables perfect topology reconstruction, despite the seeming model mismatch to real network routing.

Index Terms— Network tomography, topology, routing

1. INTRODUCTION

Network topology identification and Network tomography have gotten a lot of attention in the signal processing literature [2], and there are two complementary approaches: direct measurement using configuration files, traceroutes, or routing monitors, or indirect inference methods using measurements of packet traffic, such as delays and losses of packets [3, 7], or co-occurrences [5, 6].

Indirect inference methods have the potential to see topological structure that is hidden from direct measurements, such as where Multi-Protocol Label Switching effectively hides links and nodes from direct measurement. Despite their potential utility, indirect methods have not been widely utilized in real networks. In this paper we focus on a recently proposed method for topology inference – Network Inference from Co-Occurrences (NICO) [6]; a co-occurrence observation is an *unordered* list (or partial lists) of routers in a path.

Topology identification methods often adopt simplifying models that do not necessarily reflect the true behavior of the underlying network. The NICO inference algorithm is not an exception; it models packet routing as a Markov random walk on the network graph. While this modeling assumption facilitates inference, it is markedly at odds with the deterministic routing strategies used in the vast majority of networks. The surprising thing is that this model results in exact reconstructions for shortest-path networks. A key contribution of this paper is to resolve this apparent contradiction mathematically. We additionally show that NICO¹ will work for any network

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¹It is important to note that in [6], “NICO” refers to an EM algorithm for jointly estimating P and route-set orderings. In this paper we are using

which uses a *nested routing policy*, wherein every sub-route of a chosen route is also a route chosen by the policy. Nested routing is typical in Internet routing.

2. NOTATION

In this paper we consider a directed graph with no self-loops which represents a routing network. We define a path or route through the graph as a sequence of nodes connected by edges. We define a *route-set* or *co-occurrence* as the set of nodes along a route, where we omit the sequence or ordering of these nodes. The measurement inputs to NICO come in the form of co-occurrences.

NICO uses a Markov random walk model for routing. To do topology inference, the maximum likelihood ordering for an unordered co-occurrence is calculated from the Markov random walk probability transition matrix P . Then that ordering is used to insert nodes into the graph. Consider the link-route incidence matrix $R^{(ij)}$ for the route between nodes (i, j) , which will be a $N \times N$ binary matrix, where $R_{km}^{(ij)} = 1$ indicates that the nodes k, m occur in that sequence in the route between nodes i, j . If we take t_{ij} as the traffic from node i to node j , then the traffic weighted average of these matrices $\sum_{ij} t_{ij} R^{(ij)} = D$, a matrix of the link loads between nodes where there is a link. From D we can derive a row-stochastic transition matrix P for the whole network by normalizing each row of the matrix.

Figure 1a shows a simple example network with three nodes and directional link weights. Given such a network and routing, and $t_{ij} = 1$ for all node pairs, then an example of a link-route incidence matrix, and the link traffic matrix is

$$R^{(AB)} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad D = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 2 \\ 2 & 2 & 0 \end{pmatrix},$$

And thus the probability transition matrix P is

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{pmatrix}.$$

NICO to refer simply to topology inference using the maximum likelihood ordering of a route-set, given the transition matrix P or an estimate of P .

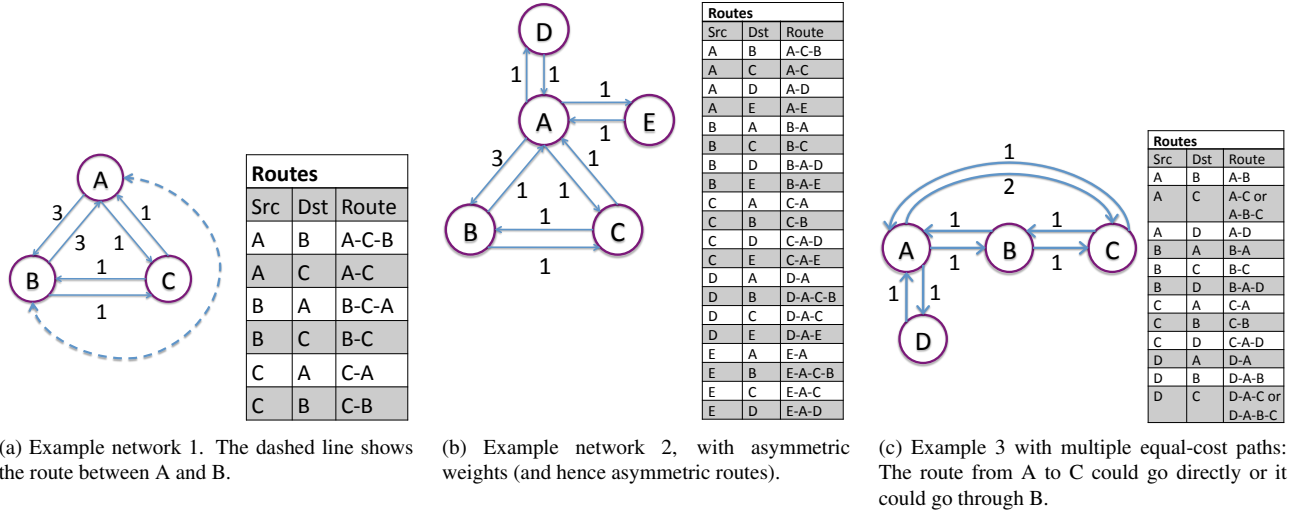


Fig. 1: Three simple examples for illustration purposes.

3. THE PROBLEM AND ITS SOLUTION

Now we can precisely define the problem of interest: given an unordered route-set for some route across a network, and the matrix P , can we find the correct ordering and reconstruct the original route? We call this the “route-ordering” problem, and it turns out the answer is yes with very general conditions. The correctly ordered route can then be used to reconstruct the topology. In this section we consider P to be known, but the simulations show that a very natural estimate of P from co-occurrence data also admits perfect route reconstructions.

Our theoretical results depend on subsets of these three conditions, which we list together for exposition purposes.

- C1.** The routers in the network of interest all use the same nested routing policy. A nested routing policy is one which will choose routes for which every subpath is also a chosen route.
- C2a.** There are no Multiple Equal-Cost (MEC) paths in the network.
- C2b.** If there are MEC paths in the network, the routing policy is shortest path and the path-cost metric is hop count. Ties must be broken in a deterministic way so that the routing remains nested.
- C3.** The first node in the true route is known. Denote it x_s .

Condition C1 is a familiar characteristic of shortest-path routing, summarized by saying “shortest paths are made up of shortest paths.” However, nested routing is more general. For instance, other types of routing such as n -stratified shortest paths (using distributive route map functions) can create nested routing policies [4]. Conditions C2a and C2b are alternative conditions. In what follows, when we refer to Condition C2, it means that either C2a or C2b must hold. Condition

C3 is natural in many problems; we are often enquiring about a route between a particular source and destination.

Definition 1 The score for an ordered route-set X_o is defined as $\prod_{(i,j) \in X_o} P_{ij}$.

In NICO, scores of this form are associated with likelihood functions of particular orderings and are used to decide which ordering to prefer. Ideally the highest score would indicate the correct ordering, though this does not hold for a generic network, even when P is known. We denote the submatrix of P corresponding to the nodes of a co-occurrence X as P_X .

Lemma 1 Assume C1 and C2, then any node permutation Π of co-occurrence X with positive score will have matrix $\Pi P_X \Pi^T$ in lower Hessenberg form with zeros on the diagonal.

The reader is encouraged to find the proof in [1]; we have omitted it here because of space restrictions. The implication is that, for some permutation Π , if the matrix $\Pi P_X \Pi^T$ is not lower Hessenberg, then the score of the ordering captured by Π is zero. We next consider the number of possible permutations resulting in a lower-Hessenberg matrix, and show that there are at most 2^{n-1} possible orderings of the length- n route with a positive score, rather than the $n!$ that we might naively expect.

Lemma 2 Under C1 and C2, there are at most 2^{n-1} permutations of a co-occurrence X that have a non-zero score.

Again we have omitted the proof to save space; refer to [1]. For an illustration of how the proof works, see Figure 2. Now we have all the pieces to put together our main result, expressed in Theorem 1. Adding condition C3 – knowledge of the source node – restricts the number of permutations with nonzero score to just one.

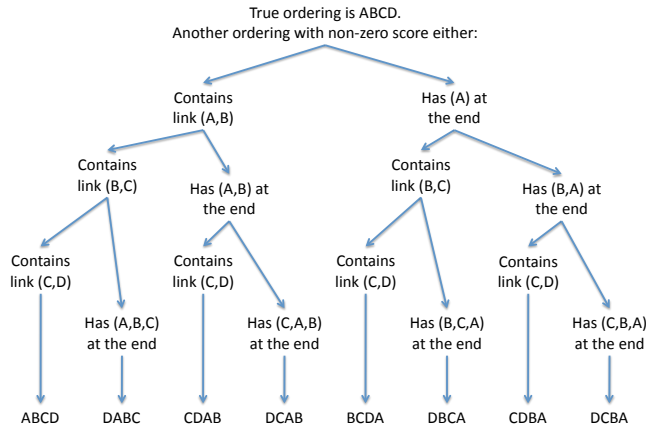


Fig. 2: Example showing that there are 2^{n-1} possible reorderings of a given route-set.

Theorem 1 Under assumptions C1-C3, and given a route-set $X = [x_1, \dots, x_n]$, an ordering $\pi = [s, \dots]$ corresponds to an ordering for a correct route in the network if and only if the score of the route $\{x_{\pi(1)}, x_{\pi(2)}, \dots, x_{\pi(n)}\}$ according to the matrix P is nonzero.

Figure 2 illustrates the result – the only way to keep $x_s = A$ at the first position is to follow the leftmost path.

Discussion: The results above are not as straight forward as they might appear. Consider a clique with unit weights. All the routes will consist of one hop paths, and the matrix P would have no zeros, so it will not help us with route-ordering. However, because the network is fully connected, all route-sets will have a single pair of nodes, and so the correct ordering is obvious. As the paths get longer, the more zeros in P allow us to resolve the order despite the factorial growth in the number of permutations.

These theorems and their proofs [1] are also predicated on knowledge of P . In practice P must be estimated from co-occurrence data; our simulations show that if we estimate D_{ij} simply by counting the number of times nodes i and j co-occur, and then normalize to get P , we get exact reconstruction.

4. EXAMPLES AND SIMULATIONS

To see more clearly what is meant by the theory, we will present a few examples, starting with the one illustrated in Figure 1a. Consider the route set $\{A, B, C\}$. There are six permutations of this route set, and Table 1 lays out these possibilities, along with a calculation of the product of the corresponding entries of P .

In Example 1 all of the scores are zero except for the correct path and its reverse. This happens when routing is symmetric. Other paths use links that are never seen in the real network. If the routing weights are not symmetric, many more possible orderings will have positive score. In fact, the

Ordering	Score	
	Example 1	Example 2
A-B-C	$0 \times 1 = 0$	$0 \times 1/4 = 0$
A-C-B	$1 \times 1/2 = 1/2$	$3/7 \times 4/7 = 12/49$
B-A-C	$0 \times 1 = 0$	$3/4 \times 3/7 = 9/28$
B-C-A	$1 \times 1/2 = 1/2$	$1/4 \times 3/7 = 3/28$
C-A-B	$1/2 \times 0 = 0$	$3/7 \times 0 = 0$
C-B-A	$1/2 \times 0 = 0$	$4/7 \times 3/4 = 3/7$

Table 1: The score calculations of the different orderings of the route set $\{A, B, C\}$ for Examples 1 and 2.

permutation with highest score is not necessarily the correct permutation. To see this, let us examine Example 2 shown in Figure 1b.

$$D = \begin{pmatrix} 0 & 0 & 6 & 4 & 4 \\ 3 & 0 & 1 & 0 & 0 \\ 3 & 4 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad P = \begin{pmatrix} 0 & 0 & \frac{3}{7} & \frac{4}{14} & \frac{4}{14} \\ \frac{3}{4} & 0 & \frac{1}{4} & 0 & 0 \\ \frac{3}{7} & \frac{4}{7} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

The resulting scores for the co-occurrence $\{A, B, C\}$ are shown in the third column of Table 1. Note that now the most likely route for route-set $\{A, B, C\}$ is C-B-A, which does not occur at all in this network. The critical feature of this example which pushes the score of a non-existent route above the scores of real routes is the asymmetry in the routes along with a heavily traversed link from B-A, which gives a high transition probability to B-A.

However again in this example, the only ordering with non-zero score which begins with the source node is the correct ordering. It is quite remarkable that this holds true in general.

The existence of multiple equal-cost (MEC) paths in the network causes a problem for us, and we see why in Figure 1c. If the traffic was split evenly between the two equal-cost paths, we would have

$$D = \begin{pmatrix} 0 & 3 & 1 & 3 \\ 2 & 0 & 2 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 0 & 0 & 0 \end{pmatrix}, \quad P = \begin{pmatrix} 0 & 3/7 & 1/7 & 3/7 \\ 1/2 & 0 & 1/2 & 0 \\ 2/3 & 1/3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$

If the co-occurrence were $\{A, B, C\}$, no permutation gives a lower Hessenberg submatrix, and the most likely ordering would be C-A-B with score $2/3 \times 3/7 = 2/7$, as opposed to the correct A-B-C, which has score $3/7 \times 1/2 = 3/14$.

4.1. Simulation Results

These simulations show the performance of NICO, using the true transition matrix P , and an estimate based on empirical frequencies of co-occurrence. We used simulated Rocketfuel [8] topologies. These unexpectedly accurate results were our motivation for developing the theory in Section 3.

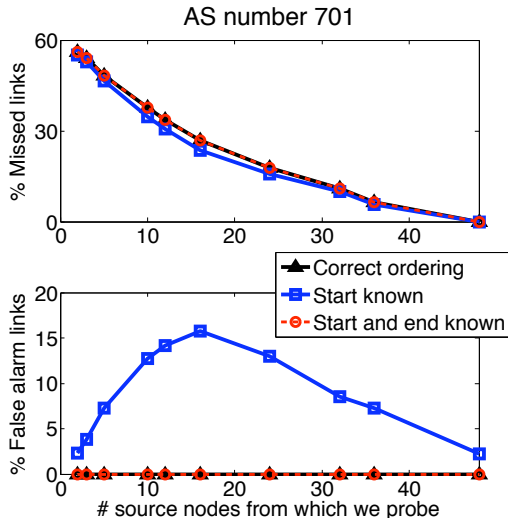


Fig. 3: A Rocketfuel topology with average degree ≈ 8 .

Figure 3 shows the results on Rocketfuel topology 701, with 48 nodes and 368 links. All destination nodes are probed, and the results are shown as we increase the number of source nodes, thus increasing the number of measurements. Even the best method cannot see the entire network when some links are not probed; thus there are missed links even in the best case, which is shown by the curve for correct orderings. The other two curves are using an empirical estimate of P calculated directly from the co-occurrence frequencies: the entry D_{ij} was found by counting the number of times nodes i and j showed up together in a route-set, and then P is the normalized version of D . We used this empirical estimate of P along with knowledge of the source only, and then we used it along with knowledge of both the source and destination. In the former case, we saw mistakes in the ordering of end nodes; this actually results in fewer missed links but many more false alarms. In the latter case, we achieved the best possible reconstruction of the network.

When the true P is used, as described in the proofs, as expected we get the best possible performance. The results when all sources are used for probing can be seen in Table 2 for eight Rocketfuel topologies. The results are perfect except for a few false alarms when only the source is known. The results are the same even when only two sources are used to probe: All methods reach the best possible performance except for the method which uses the empirical P and the source only. We refer the reader to [1] for a much more detailed set of simulation results.

5. CONCLUSION

Network inference from co-occurrence measurements, using maximum likelihood and a Markov random walk model for routing, results in perfect reconstruction of shortest-path

Rocketfuel Topology #	True P		Src only		Src&Dst	
	FA	M	FA	M	FA	M
1239	0	0	0.75	0	0	0
7018	0	0	2.80	0	0	0
70181	0	0	2.61	0	0	0
3561	0	0	2.05	0	0	0
1	0	0	0.19	0	0	0
701	0	0	2.24	0	0	0
2914	0	0	0.93	0	0	0
3356	0	0	5.78	0	0	0

Table 2: Missing links (M) and false alarm (FA) links as a percentage of total links, when all source nodes were used for probing. Note the perfect reconstruction attained when the source and destination are known. A small number of false positives occur when only the source is known.

topologies. We have provided a thorough foundation for further study of the connection between a Markov random walk routing model and real routes resulting from nested routing policies. The connection is surprising but fundamental, and will lead to improved network measurement analysis.

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